# INTRODUCTION TO RENDERING TECHNIQUES

#### What is 3D Graphics?

#### Why 3D?



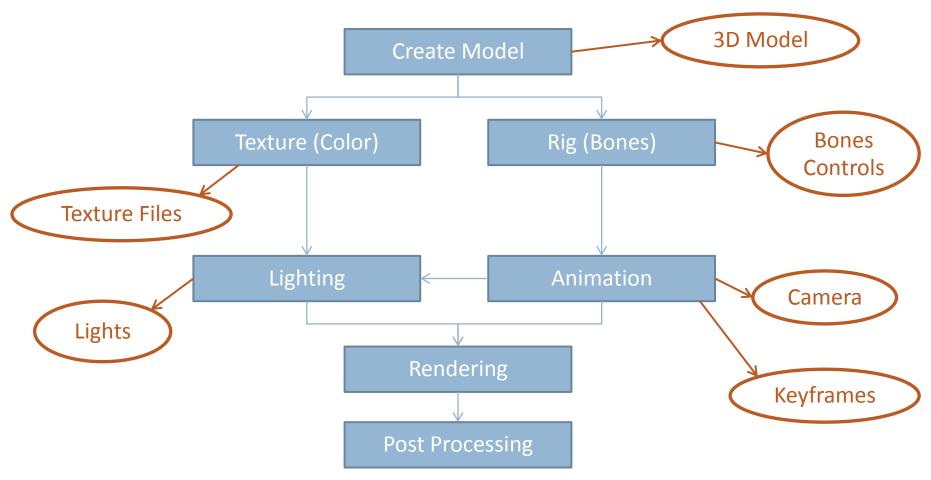
- Draw one frame at a time
- X 24 frames per second
- 150,000 frames for a feature film
- Realistic rendering is hard
- Camera movement is hard
- Interactive animation is hard



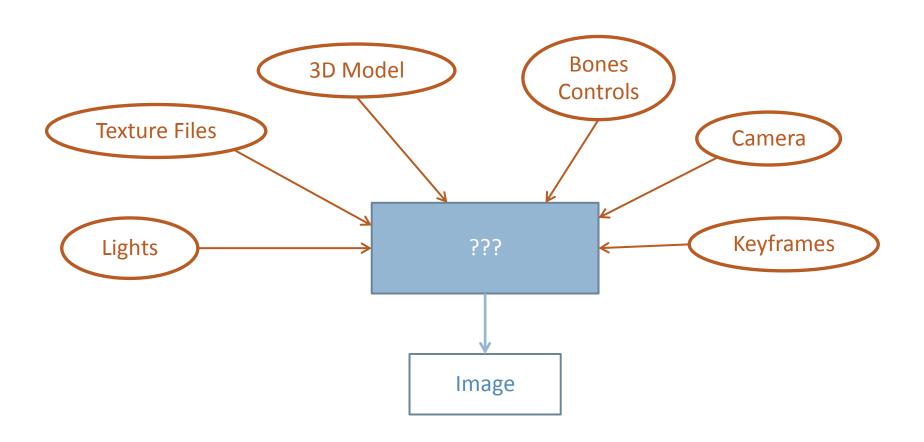
- Model only once
- Color / texture only once
- Realism / hyper realism
- A lot of reuse
- Computer time instead of artists time
- Can be interactive (games)

### What is 3D Graphics?

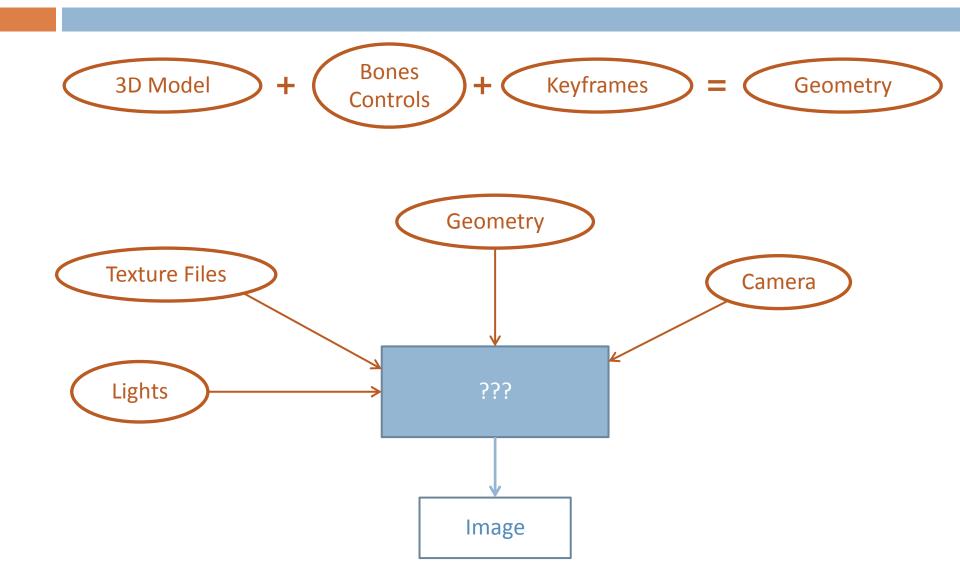
Artists workflow – in a nutshell



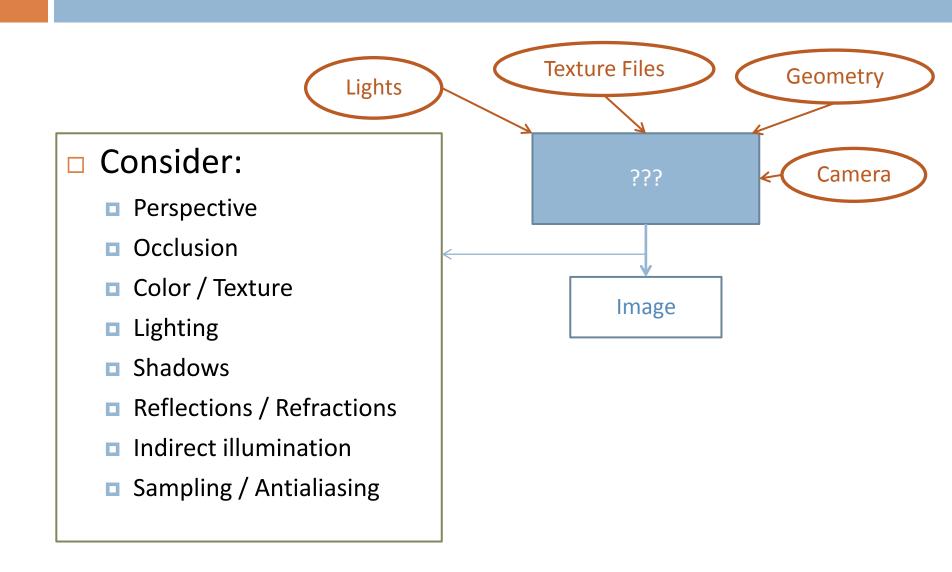
# What is Rendering?



# What is Rendering?



### What is Rendering?



#### Two Approaches

- Start from geometry
  - For each polygon / triangle:
    - Is it visible?
    - Where is it?
    - What color is it?

Rasterization

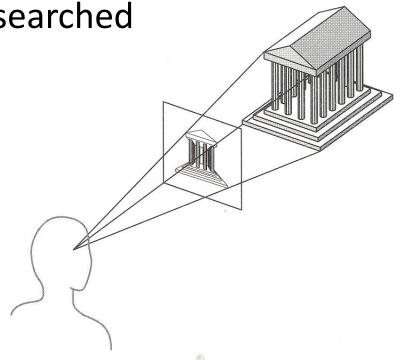
- Start from pixels
  - For each pixel in the final image:
    - Which object is visible at this pixel?
    - What color is it?

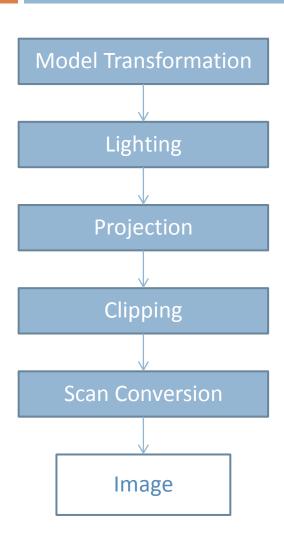
**Ray Tracing** 

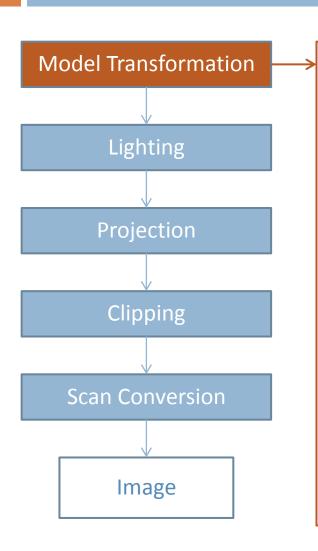
#### RASTERIZATION

#### Rasterization

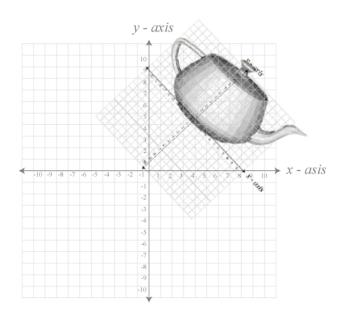
- Basic idea: Calculate projection of each triangle onto the 2D image space
- Extensively used and researched
- Optimized by GPU
- Strongly parallelized
- OpenGL
- DirectX

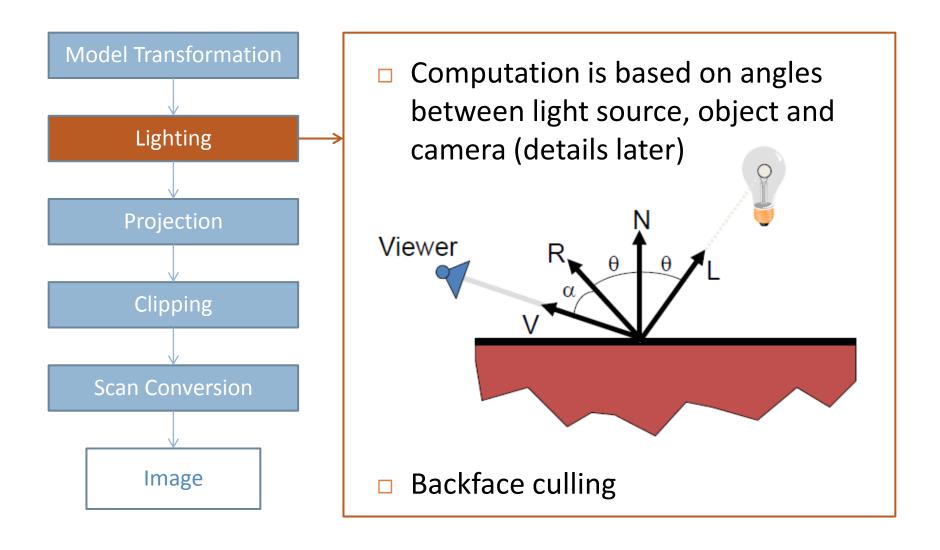


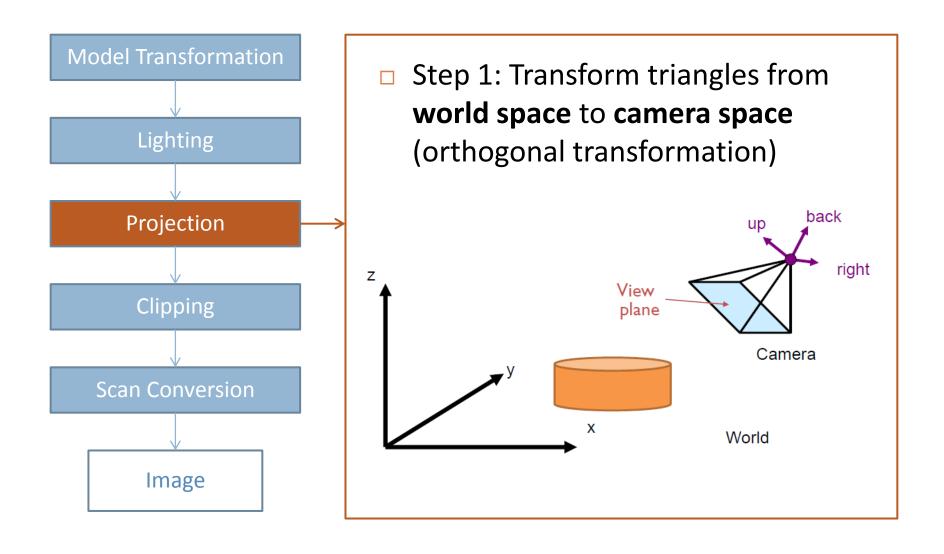


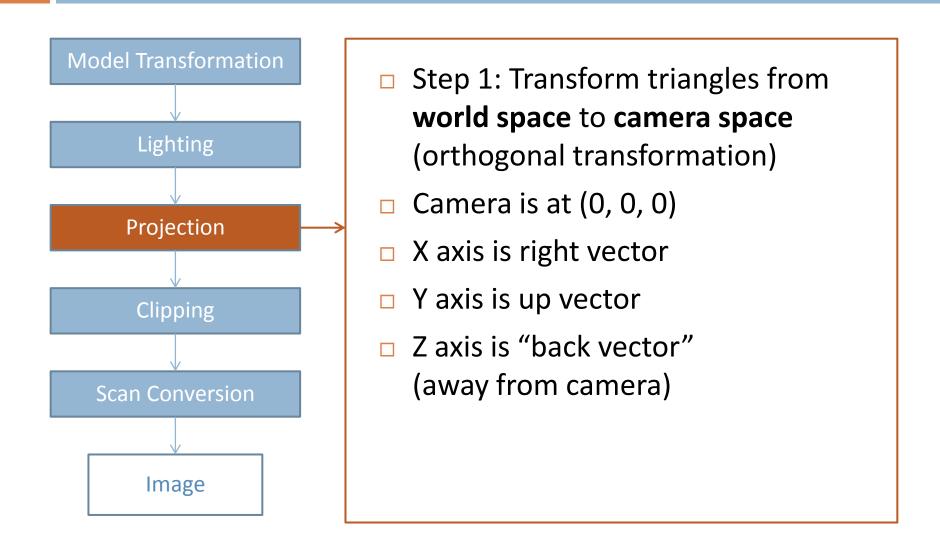


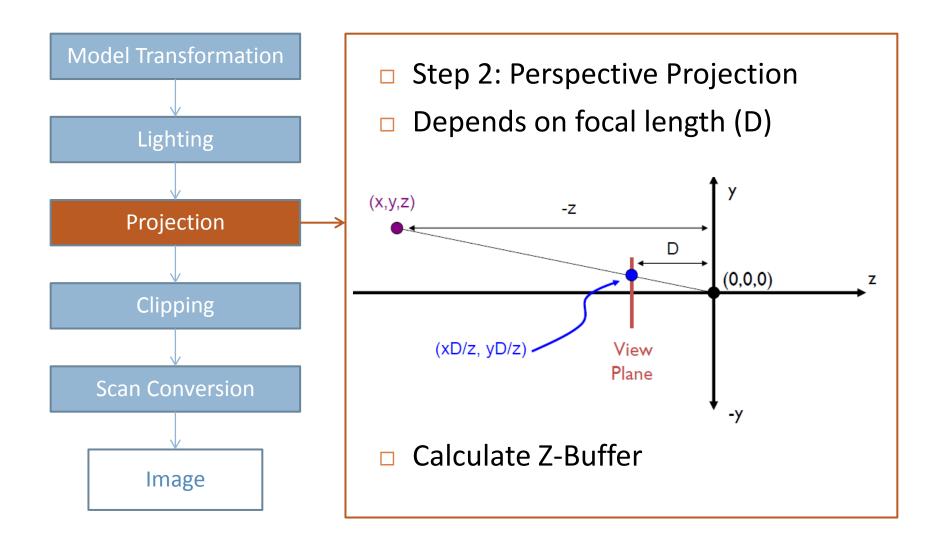
- Transform each triangle from object space to world space
- Local space -> Global space

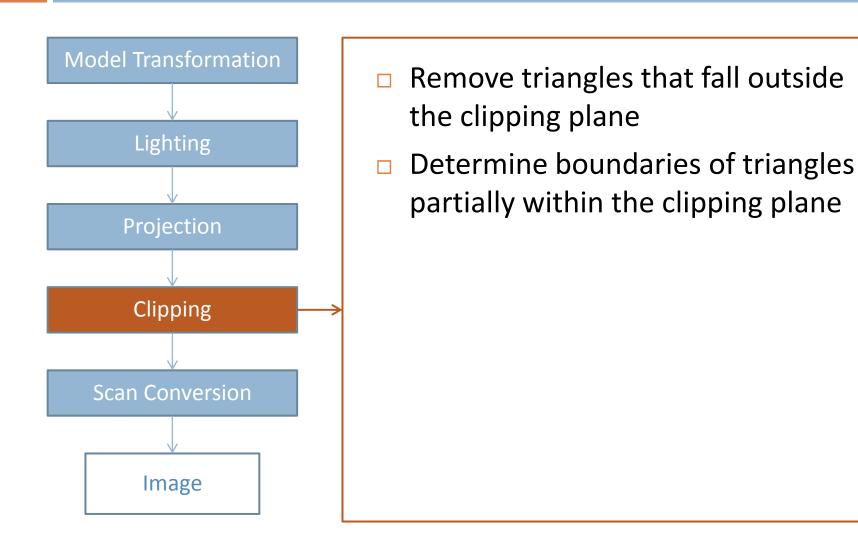


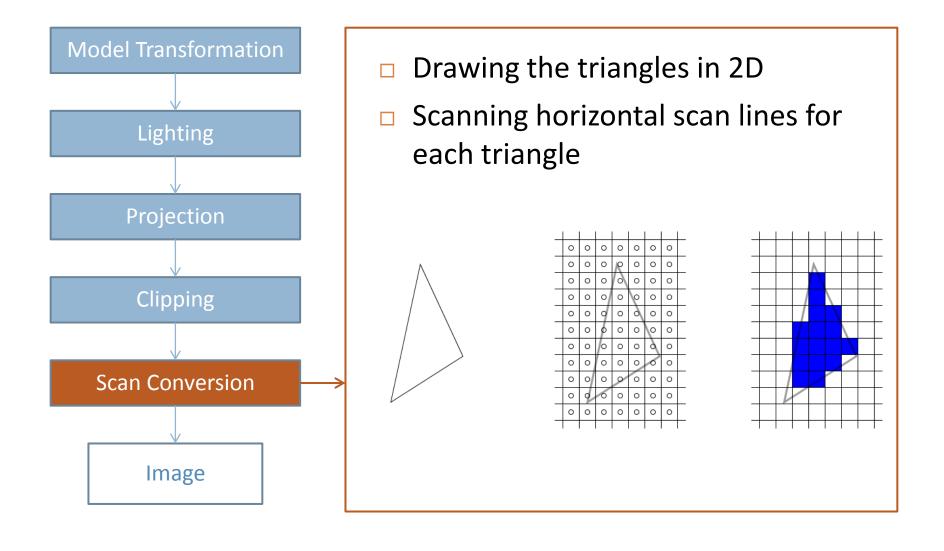


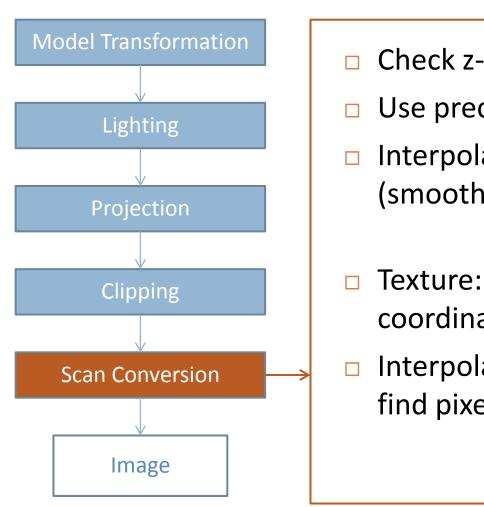








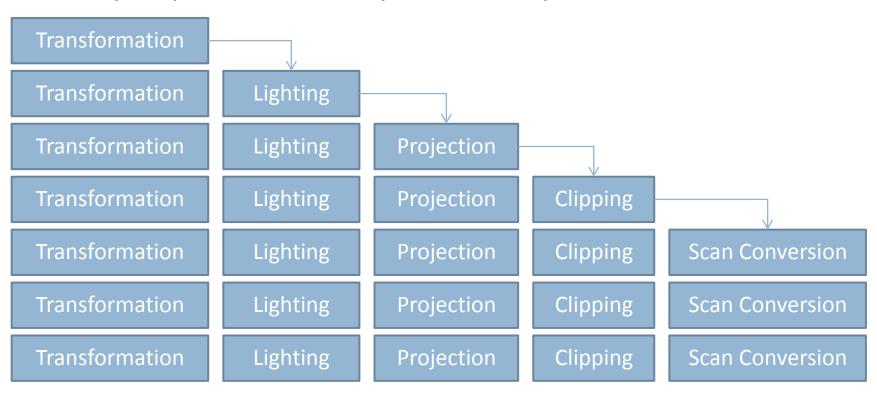




- Check z-buffer for intersections
- Use precalculated vertex lighting
- Interpolate lighting at each pixel (smooth shading)
- Texture: Every vertex has a texture coordinate (u, v)
- Interpolate texture coordinates to find pixel color

#### Rasterization – Parallel Processing

- Triangles are independent except for z-buffer
- Every step is calculated by a different part in the GPU

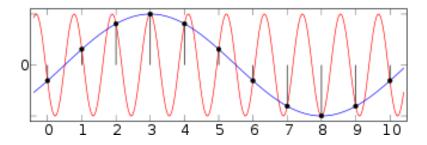


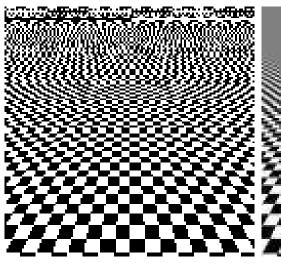
#### Rasterization – Parallel Processing

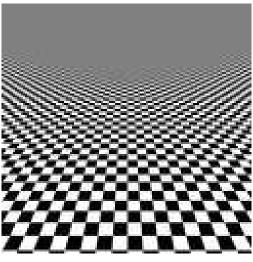
- Modern GPUs can draw 600M polygons per second
- Suitable for real time applications (gaming, medical)
- But what about...
  - Shadows?
  - Reflections?
  - Refractions?
  - Antialiasing?
  - Indirect illumination?

# Rasterization – Antialiasing

#### Aliasing examples



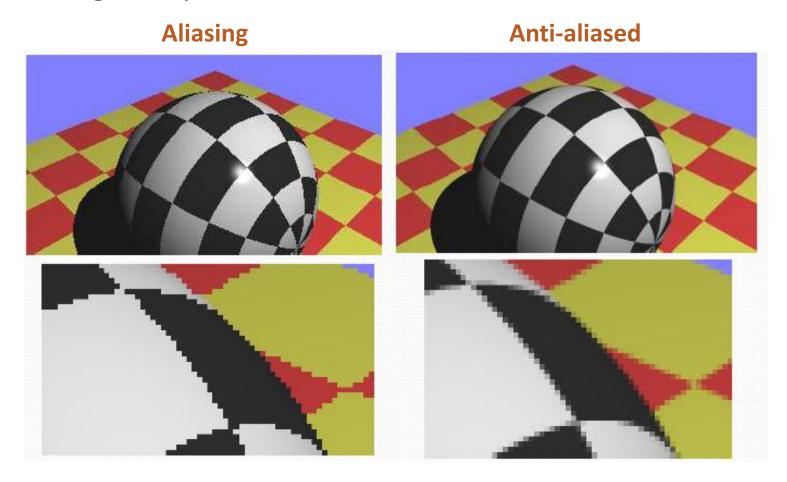






# Rasterization – Antialiasing

#### Aliasing examples



#### Rasterization – Antialiasing

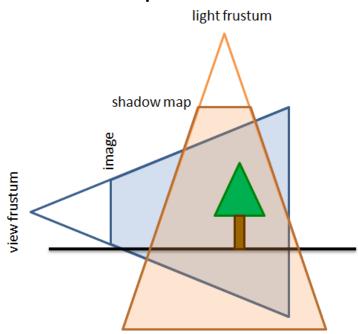
- Antialiasing: Trying to reduce aliasing effects
- Simple solution: Multisampling
- Only the last step changes!
- During scan conversion, sample subpixels and average



- This is equivalent to rendering a larger image
- Observation: Rendering twice larger resolution costs less then rendering twice – since scanline is efficient and the rest doesn't change!

#### Rasterization – Shadow Maps

- Render an image from the light's point of view (the light is the camera)
- **Shadow map**
- Keep "depth" from light of every pixel in the map
- During image render:
   Calculate position and depth on the shadow map for each pixel in the final image (not vertex!)
- If pixel depth > shadow map depth the pixel will not receive light from this source

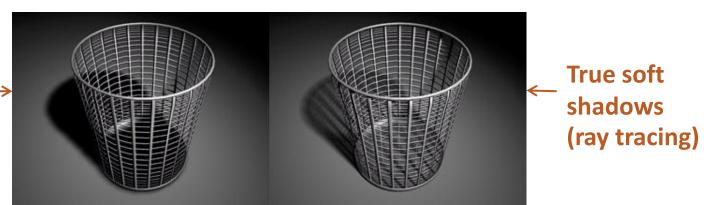


#### Rasterization – Shadow Maps

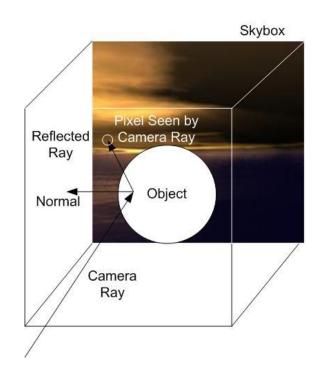
- This solution is not optimal
- Shadow map resolution is not correlated to render resolution – one shadow map pixel can span a lot of rendered pixels!
- Shadow aliasing
- Only allows sharp shadows
- Semi-transparent objects

Various hacks and complex solutions

Blurred hard \_\_ shadows (shadow map)



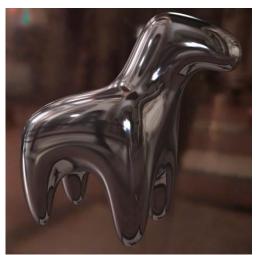
- Not a true reflection a "cheat"
- Precalculate reflection map from a point in the center (can be replaced by an existing image)
- The reflection map is mapped to a sphere or cube surrounding the scene
- Each direction (vector) is mapped to a specific color according to where it hits the sphere / cube
- During render, find the reflection color according to the reflection vector of each pixel (not vertex!)



- Can produce fake reflections (no geometry needed)
- Works well for:
  - Environment reflection (landscape, outdoors, big halls)
  - Distorted reflections
  - Weak reflections (wood, plastic)
  - Static scenes
- Not so good for:
  - Reflections of near objects
  - Moving scenes
  - Mirror like objects
  - Optical effects

Examples: Reflection maps





Used to create the map





Examples: Ray traced reflections



#### Examples:



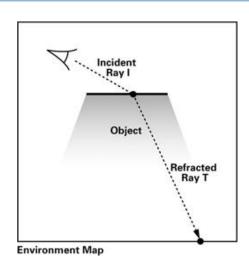
**Reflection Map** 

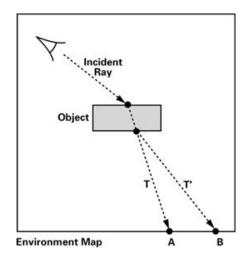


**Ray Traced Reflection** 

#### Rasterization – Refractions

- There is no real solution
- Refraction maps: same as reflection maps but the angle is computed using refractive index
- Only simulates the first direction change, not the second (that would require ray tracing)
- Refraction is complex so fake refractions are hard to notice
- Doesn't consider near objects, only static background



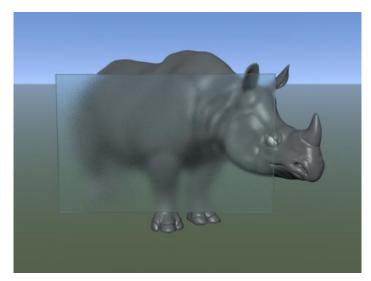


#### Rasterization – Refractions

- Other "fake" solutions:
- Distort the background according to a precomputed map
- "Bake" ray traced refractions into a texture file (for static scenes)



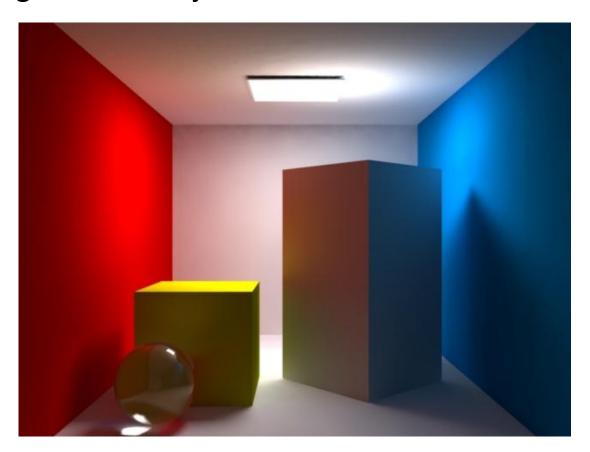
**Refraction Map** 



**Distort Background** 

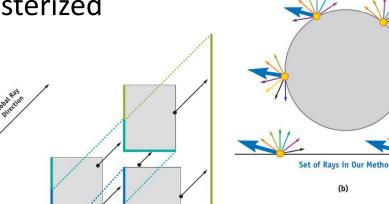
#### Rasterization – Indirect Illumination

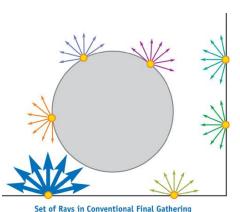
 Indirect / global illumination means taking into account light bouncing off other objects in the scene



#### Rasterization – Indirect Illumination

- Surprisingly, there are methods to approximate global illumination using only rasterization, without ray tracing
- "High-Quality Global Illumination Rendering Using Rasterization", Toshiya Hachisuka, The University of Tokyo
- Main idea: Use a lot of fast rasterized "renders" from different angles to compute indirect illumination at each point
- Rasterization is super quick on GPU





#### Rasterization – Indirect Illumination

#### Results:

#### Results of equal render time



Photon mapping (ray tracing)



Rasterizer (GPU)



#### TRANSFORMATIONS

#### **Transformations**

- We saw 2 types of transformations
- Viewing transformation: Can move, rotate and scale the object but does not skew or distort objects
- Perspective projection: This special transformation projects the 3D space onto the image plane
- How do we represent such transformations?
- □ Homogeneous coordinates: Adding a 4<sup>th</sup> dimension to the 3D space

#### Types of transformations

#### Scale

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

#### **Translate (move)**

Translate (move)
$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & 1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

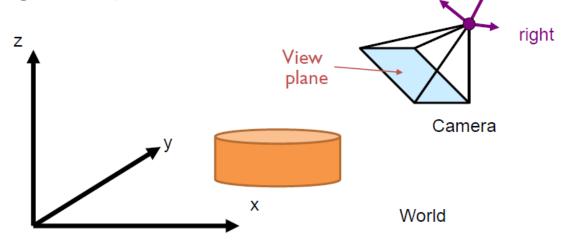
#### Rotations

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

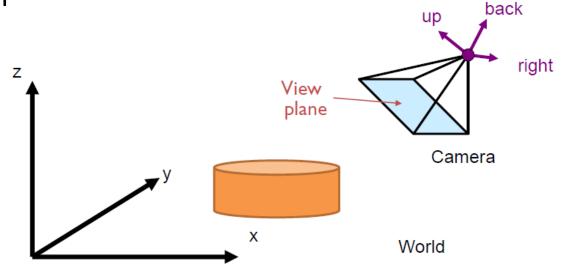
- Any combination of these matrices is a viewing transformation matrix
- Last coordinate is only for moving the pivot,
   w' is always 1 and will not be used
- How to find the transformation to a certain view (could be camera, light, etc)?



back

up

- After the transformation:
- Eye position should be at (0, 0, 0)
- X axis = right vector
- Y axis = up vector
- Z axis = back vector



 It is easy to construct the invert transformation, from camera coordinates to world

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} R_x & U_x & B_x & E_x \\ R_y & U_x & B_y & E_y \\ R_z & U_z & B_z & E_z \\ R_w & U_w & B_w & E_w \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

#### **Examples:**

(0, 0, 0) -> Eye Position

$$\begin{bmatrix} E_{x} \\ E_{y} \\ E_{z} \\ E_{w} \end{bmatrix} = \begin{bmatrix} R_{x} & U_{x} & B_{x} & E_{x} \\ R_{y} & U_{x} & B_{y} & E_{y} \\ R_{z} & U_{z} & B_{z} & E_{z} \\ R_{w} & U_{w} & B_{w} & E_{w} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Camera X Axis -> Origin + Right vector

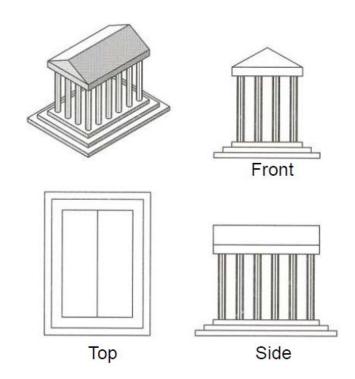
$$\begin{bmatrix} E_x \\ E_y \\ E_z \\ E_w \end{bmatrix} = \begin{bmatrix} R_x & U_x & B_x & E_x \\ R_y & U_x & B_y & E_y \\ R_z & U_z & B_z & E_z \\ R_w & U_w & B_w & E_w \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} R_x + E_x \\ R_y + E_y \\ R_z + E_z \\ R_w + E_w \end{bmatrix} = \begin{bmatrix} R_x & U_x & B_x & E_x \\ R_y & U_x & B_y & E_y \\ R_z & U_z & B_z & E_z \\ R_w & U_w & B_w & E_w \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Now all we have to do is invert T (always invertible), and we have our view transformation

### **Projections**

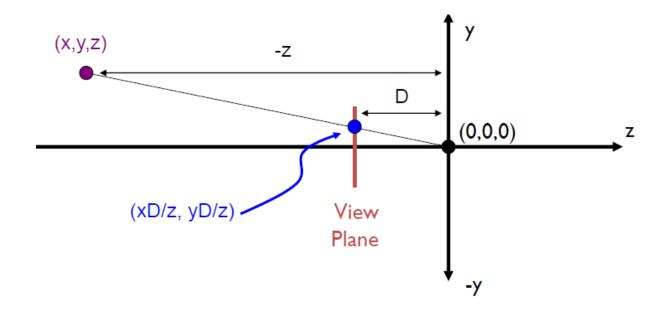
- A projection transform points from higher dimension to a lower dimension, in this case 3D -> 2D
- The most simple projection is orthographic
- Simply remove the Z axis after the viewing transformation

$$\begin{bmatrix} x_p \\ y_p \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x_v \\ y_v \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$$



#### Perspective Projections

- Perspective projections map points onto the view plane toward the center of projection (the viewer)
- $\square$  Since the viewer is at (0, 0, 0) the math is very simple
- D is called the focal length
- $\square$  x' = x\*(D/z)
- $\neg y' = y*(D/z)$



#### Perspective Projections

 Matrix form of the perspective projection using homogeneous coordinates

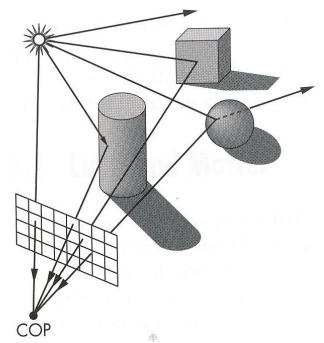
$$\begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} dx & dy & dz & z \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{d}{z}x & \frac{d}{z}y & d \end{bmatrix}$$
**Divide by 4th coordinate**
(the "w" coordinate)

- □ Singular matrix projection is many to one
- □ D = infinity gives an orthographic projection
- □ Points on the viewing plane z = D do not move
- Points at z = 0 are not allowed usually by using a clipping plane at z = ε

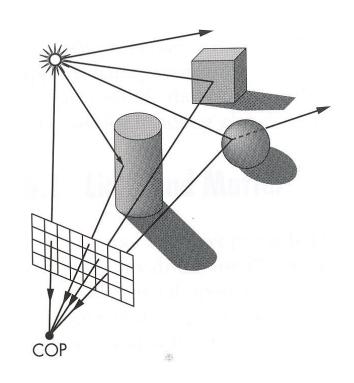
#### LIGHTING

#### RAY TRACING

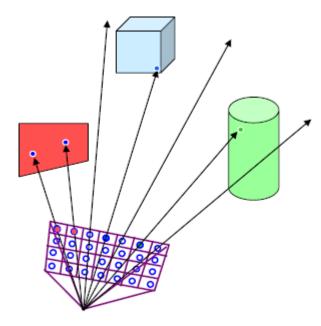
- Basic idea: Shoot a "visibility ray" from center of projection (camera) through each pixel in the image and find out where it hits
- This is actually backward tracing
   instead of tracing rays from
   the light source, we trace the
   rays from the viewer back to
   the light source



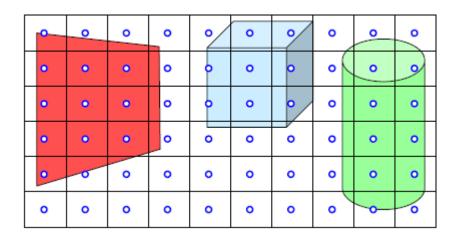
- Backward tracing is called Ray Casting
- Simple to implement
- For each ray find intersections
   with every polygon slow...
- Easy to implement realistic lighting, shadows, reflections and refractions, and indirect illumination



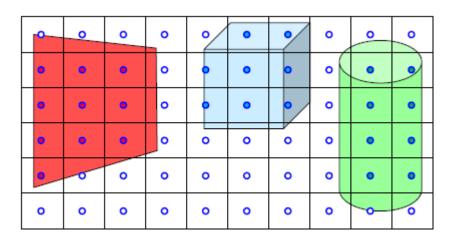
- For each sample (pixel or subpixel):
- Construct a ray from eye position through viewing plane



- □ For each sample (pixel or subpixel):
- Construct a ray from eye position through viewing plane
- Find first (closest) surface that intersects the ray



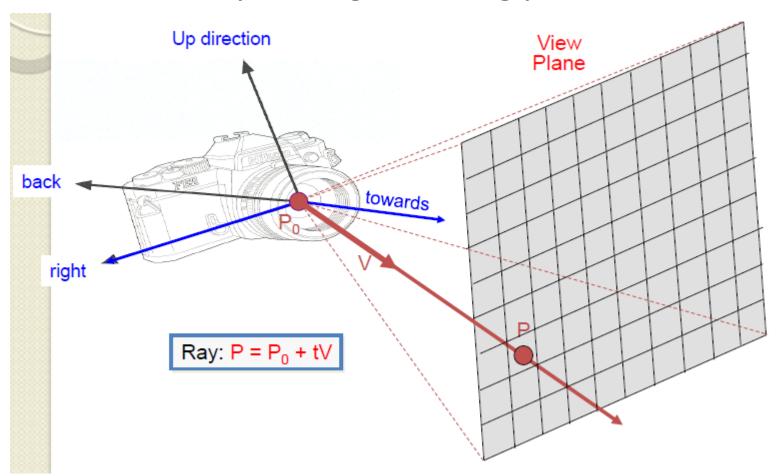
- □ For each sample (pixel or subpixel):
- Construct a ray from eye position through viewing plane
- Find first (closest) surface that intersects the ray
- Compute color based on surface radiance



- For each sample (pixel or subpixel):
- Construct a ray from eye position through viewing plane
- Find first (closest) surface that intersects the ray
- Compute color based on surface radiance
- Computing radiance requires casting rays toward the light source, reflected and refracted objects and recursive illumination rays from reflected and refracted objects

# Ray Tracing – Casting Rays

Construct a ray through viewing plane:



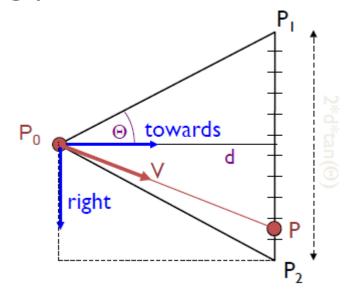
# Ray Tracing – Casting Rays

- Construct a ray through viewing plane:
- 2D Example:

$$P_1 = P_0 + d*towards - d*tan(\Theta)*right$$
  
 $P_2 = P_0 + d*towards + d*tan(\Theta)*right$ 

$$P = P_1 + (i/width - 0.5) * 2*d*tan (\Theta)*right$$
  
 $V = (P - P_0) / ||P - P_0||$ 

For every i between (-width/2) and (width/2)

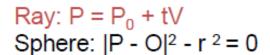


Ray: 
$$P = P_0 + tV$$

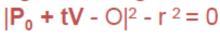
- Finding intersections
  - Intersecting spheres
  - Intersecting triangles (polygons)
  - Intersecting other primitives
  - Finding the closest intersection in a group of objects / all scene

Finding intersections with a sphere: Algebraic method

P'



Substituting for P, we get:



Solve quadratic equation:

$$at^2 + bt + c = 0$$

where:

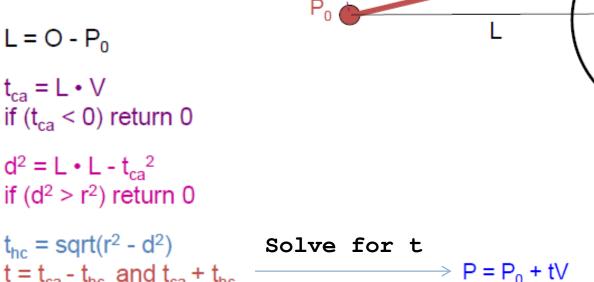
$$a = 1$$
  
 $b = 2 \text{ V} \cdot (P_0 - O)$  Solve for t  
 $c = |P_0 - O|^2 - r^2 = 0$   $\longrightarrow$   $P = P_0 + tV$ 

Finding intersections with a sphere:

Geometric method

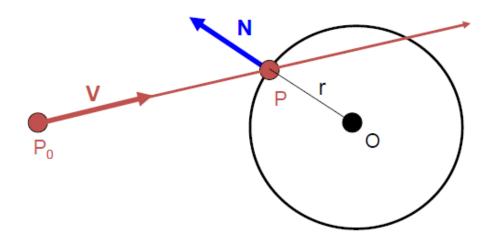
Ray: 
$$P = P_0 + tV$$
  
Sphere:  $|P - O|^2 - r^2 = 0$   
 $L = O - P_0$   
 $t_{ca} = L \cdot V$   
if  $(t_{ca} < 0)$  return 0  
 $d^2 = L \cdot L - t_{ca}^2$   
if  $(d^2 > r^2)$  return 0

 $t = t_{ca} - t_{hc}$  and  $t_{ca} + t_{hc}$ 

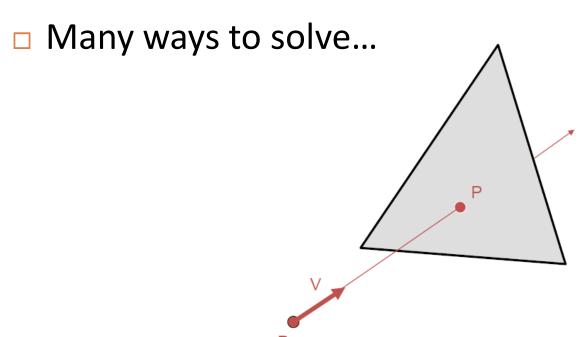


- Finding intersections with a sphere:Calculating normal
- We will need the normal to compute lighting, reflection and refractions

$$N = (P - O) / ||P - O||$$



- Finding intersections with a triangle:
- Step 1: find intersection with the plane
- Step 2: check if point on plane is inside triangle



Step 1: find intersection with the plane: Algebraic method

Ray:  $P = P_0 + tV$ 

Plane:  $N(P-P_0)=0 \rightarrow P \cdot N + c = 0$ 

Substituting for P, we get:

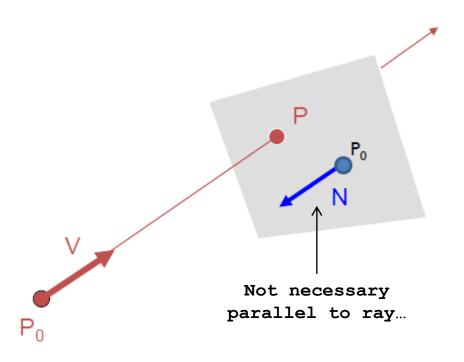
$$(P_0 + tV) \cdot N + c = 0$$

Solution:

$$\mathsf{t} = -(\mathsf{P}_0 \bullet \mathsf{N} + c) \, / \, (\mathsf{V} \bullet \mathsf{N})$$

And the intersection at:

$$P = P_0 + tV$$



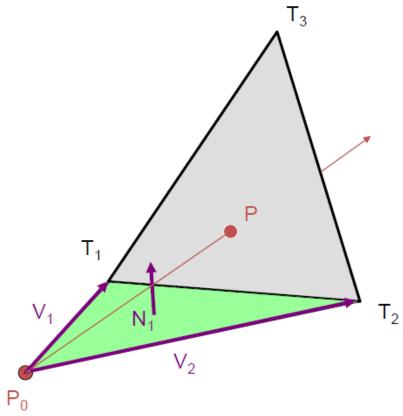
Step 2: Check if point is inside triangle
 Algebraic method

```
For each side of triangle
```

```
V_1 = T_1 - P_0
V_2 = T_2 - P_0
N_1 = V_2 \times V_1
Normalize N_1
if (P - P_0) \cdot N_1 < 0
return FALSE;
```

If all 3 succeed the point is inside the triangle

end



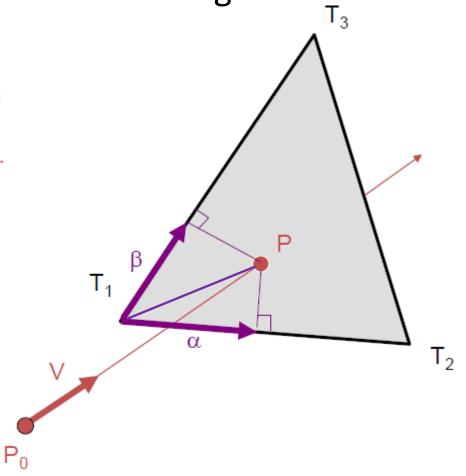
Step 2: Check if point is inside triangle
 Paramteric method

Compute 
$$\alpha$$
,  $\beta$ :  

$$P = \alpha (T_2 - T_1) + \beta (T_3 - T_1)$$

Using dot products  $(P-T_1) \bullet (T_2-T_1)$  and  $(P-T_1) \bullet (T_3-T_1)$ 

Check if point inside triangle.  $0 \le \alpha \le 1$  and  $0 \le \beta \le 1$   $\alpha + \beta \le 1$ 



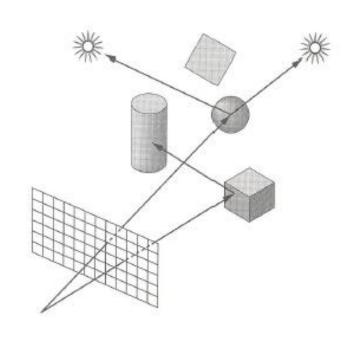
- Ray tracing can support other primitives
  - Cone, Cylinder, Ellipsoid: similar to sphere
  - Convex Polygon:
     Point in Polygon is a basic problem in computational geometry and has algebraic solutions
  - Concave Polygon:
     Same plane intersection
     More complex point-in-polygon test
  - Alternatively, divide the polygon to triangles and check each triangle

- Find closest intersection:
- Simple solution is go over each polygon in the scene and test for intersections
- We will see optimizations for this later... (maybe)

We have an intersection – what now?

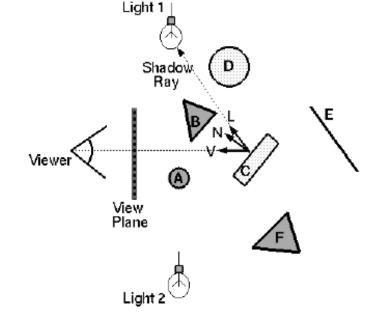
#### Ray Tracing – Computing Color

- Computing lighting can be similar to the process when rasterizing (using normals)
- This is not for a vertex but for the intersection point
- For better accuracy: ray trace lighting
  - At each intersection point cast a ray towards every light source
  - Provides lighting, shadows, reflections, refractions and indirect illumination
  - Easy to compute soft shadows, area lights



## Ray Tracing – Shadows

- Shadow term tell which light source are blocked
- $S_L = 0$  if ray is blocked,  $S_L = 1$  otherwise
- Direct illumination is only calculated for unblocked lights

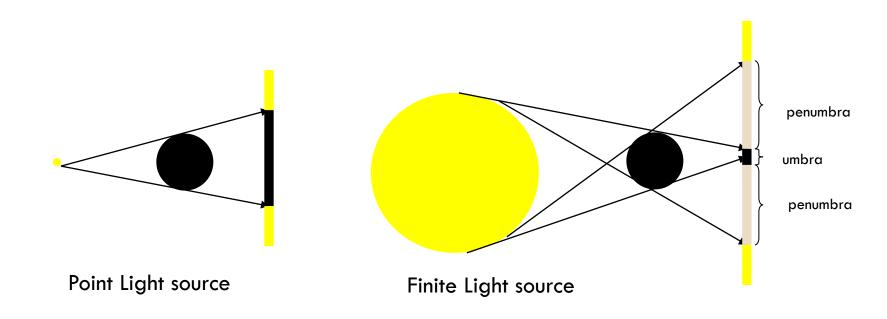


Illumination formula:

$$I = I_E + K_A I_A + \sum_L (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L$$
Shadow term

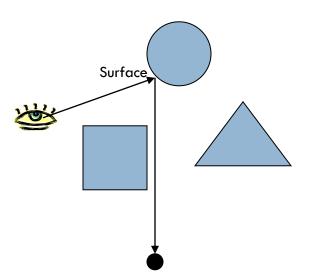
## Ray Tracing – Soft Shadows

- Why are real life shadows soft?
- Because light source is not truly a point light

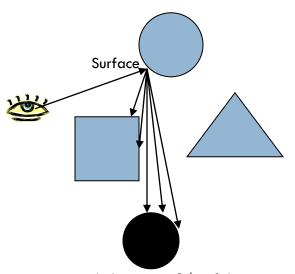


## Ray Tracing – Soft Shadows

 Simulate the area of a light source by casting several (random) rays from the surface to a small distance around the light source



Point light source: The surface is completely lighted by the light source.

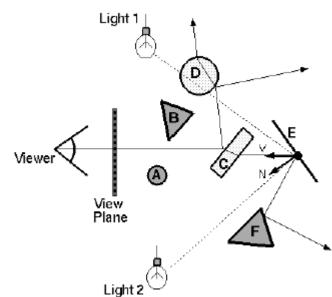


Finite light source: 3/5 of the rays reach the light source. The surface is partially lighted.

#### Ray Tracing – Reflection / Refraction

Recursive ray tracing: Casting rays for reflections
 and refractions

 For every point there are exact directions to sample reflection and refraction (calculated from normal)

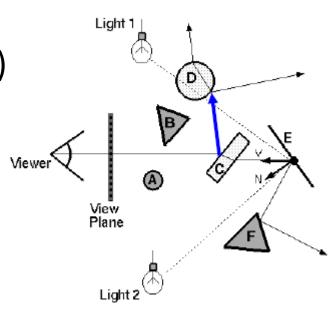


Illumination formula:

$$I = I_E + K_A I_A + \sum_{L} (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L + K_S I_R + K_T I_T$$

#### Ray Tracing – Reflection / Refraction

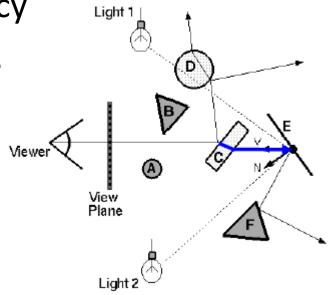
- Cast a reflection ray
- Compute color at the hit point (using ray tracing again!)
- Multiply by reflection term of the material
- To avoid aliasing sample
   several rays in the required direction and average



$$I = I_E + K_A I_A + \sum_{L} (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L + (K_S I_R + K_T I_T)$$

#### Ray Tracing – Reflection / Refraction

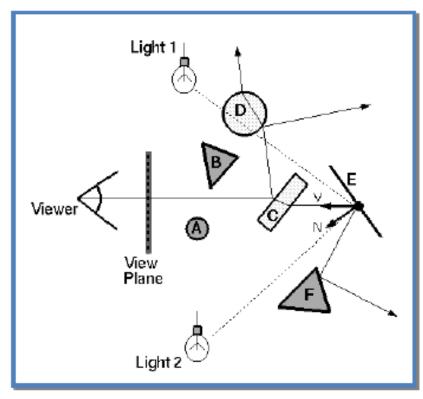
- ... And the same for refractions
- Last coefficient is transparency
- $□ K_T = 1 ext{ for translucent objects}$   $K_T = 0 ext{ for opaque objects}$
- Consider refractive index of object
- Again use several rays to avoid aliasing



$$I = I_E + K_A I_A + \sum_{L} (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L + K_S I_R + K_T I_T$$

#### Ray Tracing – Reflection / Refraction

Ray tree represents recursive illumination computation



Primary Ray Transmission Reflection Ray Reflection Transmission Ray Reflection Background Background Reflection Background

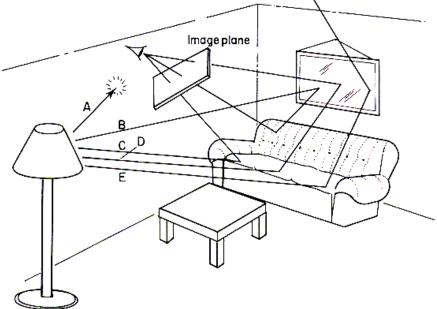
Ray traced through scene

Ray tree

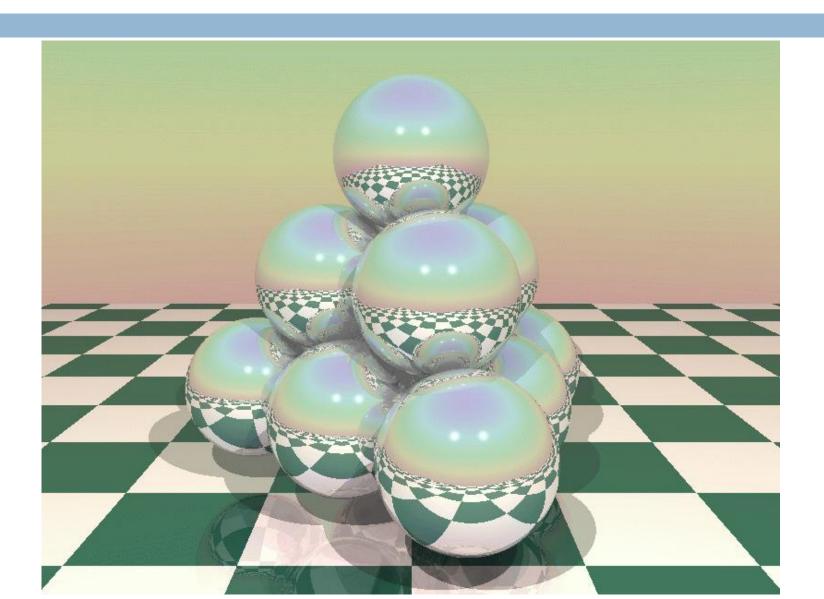
#### Ray Tracing – Reflection / Refraction

- Number of rays grows exponentially for each level!
- Common practice: limit maximum depth

 After 2-3 bouncing reflections, the cost is high and there is little benefit



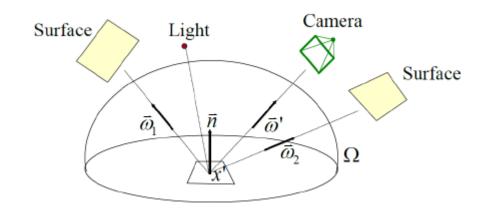
# Ray Tracing – Antialiasing



## Ray Tracing – Antialiasing

- Aliasing in ray tracing can be severe, since only one ray is casted per pixel
- The computation is based on the size of the pixels, not on the size of the actual polygons which can be relatively small
- Supersampling: Instead of casting one ray per pixel, cast several per pixel
- Since this is done at the first step, it is as inefficient as possible (running the whole process again)

- What we've seen so far is only an approximation of real lighting: The rays are only casted directly towards the light
- Use reflections, but not indirect lighting
- Global illumination: A method to approximate indirect lighting from every direction



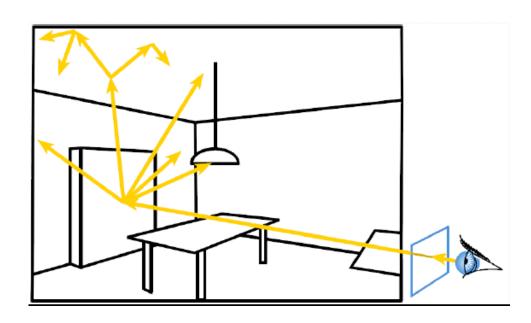
- Example:
- Top image uses direct lighting only

- Bottom image uses indirect illumination
- Notice the ground is "reflected" naturally on the character
- Not because of reflective material but because of lighting contribution



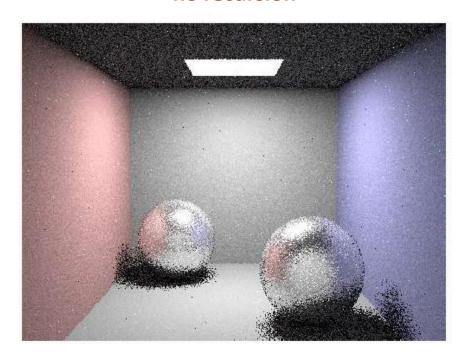
- Monte-Carlo path tracing
- Step 1: Cast regular rays through each pixel in viewing plane
- Step 2: Cast random rays from visible point
- □ Step 3: Recurse

Very expensive!

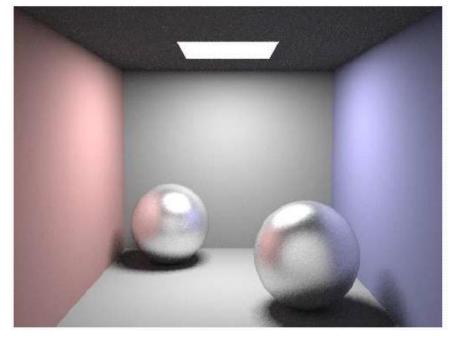


#### Monte-Carlo path tracing

1 random ray per pixel no recursion

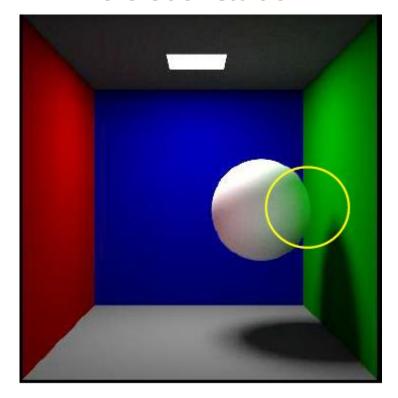


16 random rays per pixel 3 levels of recursion



- Monte-Carlo path tracing
- Need a lot of rays and recursions to look good
- Random rays cause flickering problems
- Computation time measured in hours!
- Common practice:
   Bake global illumination map of one frame and use it for all frames

64 random rays per pixel 3 levels of recursion



#### Ray Tracing - Ambient Occlusion

Ambient Occlusion is a simpler form of global illumination

Cast random rays from visible point and calculate distance

to the nearest object

 The more rays hit near objects, the point is occluded and therefore darker

- A cheat "make nice" button
- Everything looks better with ambient occlusion!

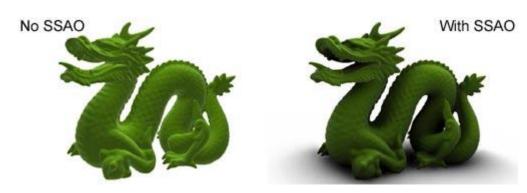


# Ray Tracing - Ambient Occlusion

Good for contact shadows

Examples:







## Summary

#### Rasterization

- Fast renderer
- Optimized for GPUs
- Antialiasing is easy and fast
- Scales well for larger images
- Parallel computing possible on GPU
- Shadows are hard to compute and inaccurate
- Relections and refractions are a hack
- Indirect illumination complex but possible (rarely used in practice)

#### **Ray Tracing**

- Slow renderer only today we see
   some real time ray tracing possible
- Not optimized for GPUs
- Antialiasing is expensive
- Doesn't scale so well
- Parallel computing is easy
- Shadows are easy including sofy shadows
- Relections and refractions are easy
- Indirect illumination complex but possible (rarely used in practice)

#### What Artists Do

- □ In practice: Both are used side by side
- Games:
   Real time, mostly rasterized except for special effects
- Movies / Animation:
  - Not real time, but time = money
     Usually a mix of rasterization and ray traced reflections / refractions.
  - Global illumination is sometimes used but usually faked using direct lights

#### What Artists Do

- Common practice: Use render layers and composite later using a video editing program (like After Effects)
- Render layers:
  - Color (radiance)
  - Reflections
  - Refractions
  - Depth map
  - Ambient Occlusion
- Makes it easy to make fast changes later without rendering again

## THAT'S ALL, FOLKS!